POLYNOMIALS "CHEAT SHEET" all math/8-19-07

Degree (highest power of the variable) (highest sum-of-exponents for multi-variable)

power	degree	name
0	0^{th}	constant
1	1^{st}	linear
2	2^{nd}	quadratic
3	3^{rd}	cubic
4	4^{th}	quartic (some books, but not all)
5 (& up)	5^{th} (& up)	none

Number of terms (monomials connected by + or – signs)

term count	terminology	name
1	1-term	monomial (product of a coefficient and variables)
2	2-term	binomial
3	3-term	trinomial
4 (& up)	4-term (& up)	none

Like terms – have same variable(s) to the same exponent(s), coefficients may differ

Adding (subtracting) – combine like terms by adding (subtracting) coefficients

$(3 + 4x) – (2 + 3x) = 3 + 4x -2 -3x = (3-2) + (4-3)x = 1 + x$

Multiplying polynomials:

```
                F  L F  L     F      O     I      L
2 binomials: FOIL   (a+bx)(c+dx) = ac + adx + bcx + bdx^2 = ac + (ad+bc)x + bdx^2
                O   I  I   O
```

```
                                      __________
                                     |______      |
                                     |        |     |
Any polynomials: rainbow (cards)    (a + bx)(c + dx)        (deal all,  to every player)
                                         |___|     |
                                         |_______ |
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Distribution (or factoring), of a common factor

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a (b + cx + dx^2)  =  ab + acx + adx^2    (works both ways)
|__|     |        |
|______|        |
|__________|                        ("a" can be any monomial,  not just a number!)
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Patterns for multiplying and factoring: (a and b may be any monomials)

$(\text{binomial})^2$ = perfect square trinomial	$(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$
(bino. sum)(bino. diff.) = diff of 2 squares	$(a + b)(a – b) = a^2 – b^2$
sum of two cubes	$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$
difference of two cubes	$a^3 – b^3 = (a – b)(a^2 + ab + b^2)$

Procedure for factoring $1x^2 + bx + c$ **(“3rd term” procedure)**

(1) list all the possible factors of the 3rd term c, positive and negative
(2) select that pair of factors that also add to form middle term coefficient b
(3) the selected factors become the constants added to x in the two binomials

example: $x^2 -7x + 10$

factors	sum
1 10	11
-1 -10	-11
2 5	7
-2 -5	-7 <<<< sums to -7, select

so $x^2 -7x + 10 = (x-2)(x-5)$

(always check by multiplying your answer to see if you get the original problem)

“Organized trial-and-error” procedure for factoring $ax^2 + bx + c$

(1) list all the possible pairs of factors of a, to be the coefficients of x
(2) list all the possible pairs of factors of c, to be the constants added to x terms
(3) multiply out all combinations of (1) and (2) until obtaining the right “bx”

example: $2x^2 +x – 3$

factors of 2 (on x^2)	factors of -3
1 and 2	-1 and 3
-1 and -2	1 and -3

combinations	result
(x-1)(2x+3)	$2x^2 +x -3$ <<<< that’s it
(-x -1)(-2x +3)	$2x^2 –x -3$
(x+1)(2x -3)	$2x^2 –x -3$
(-x+1)(-2x-3)	$2x^2 +x -3$ <<<< same as 1st one
(x+3)(2x-1)	$2x^2 +5x -3$
(-x+3)(-2x-1)	$2x^2 -5x -3$
(x-3)(2x+1)	$2x^2 -5x -3$
(-x-3)(-2x+1)	$2x^2 +5x -3$

(always check by multiplying your answer to see if you get the original problem)

Factoring by grouping:

(1) Group like terms having a common factor and factor it out
(2) Look for a common factor between your factored groups

Example: factor $5x + 5y + x^2 + xy$

Group the y's together, and the remaining x's together:
$5y + xy \quad + \quad 5x + x^2$
$y(5 + x) \quad + \quad x(5 + x)$ note (5 + x) in common
$(y + x)(5 + x)$

(always check by multiplying your answer to see if you get the original problem)

Factoring $ax^2 + bx + c$ by "the Box"

Illustrate by example: factor $8x^2 – 2x – 15$

Set up the box:

$8x^2$		$-2x$
	-15	

Multiply down diagonal $-120x^2$

List all the factors of -120x2 and find the ones that add to -2x; the list shown is the complete one, but you can often see the pattern and shortcut it:

-1x	120x	119x	
1x	-120x	-119x	
2x	-60x	-58x	
-2x	60x	58x	
3x	-40x	-37x	
-3x	40x	37x	
4x	-30x	-26x	
-4x	30x	26x	
5x	-24x	-19x	
-5x	24x	19x	
6x	-20x	-14x	
-6x	20x	14x	
8x	-15x	-7x	
-8x	15x	7x	
10x	**-12x**	**-2x**	**<<<< that's it!!!**
-10x	12x	2x	

Put the two factors in the other two box spaces (doesn't matter which):

$8x^2$	$10x$
$-12x$	-15

Pull out common factors leftward and upward, and use them as your binomials:

	$4x$	5
$2z$	$8x^2$	$10x$
-3	$-12x$	-15

Check to make sure, sometimes a +/- sign will be off in one or the other

$$(2x - 3)(4x + 5) = 8x^2 -12x +10x -15 \ ?$$
$$= 8x^2 -2x -15 \quad \text{VERIFIED}$$

Use this procedure whenever there are multiple sets of factors for the a and the c in the original quadratic. It will save considerable time and effort. Use direct trial and error whenever there is only one set of factors for the a and the c.